DETERMINATION OF THE TEMPERATURE FIELD IN COMPOSITE TWO-DIMENSIONAL SYSTEMS

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In composite two-dimensional multilayer systems the temperature field is determined by introducing effective thermophysical characteristics for heterogeneous media.

The design and operation of electric and power facilities are impossible without determination of unsteady thermal fields in heterogeneous systems. For calculation of one-dimensional layered structures there are reliable precise analytical [1] and approximate methods, the latter based on substitution of heterogeneous systems by quasihomogeneous ones [2-5]. Two-dimensional systems are usually analyzed on computers [6], and their analytical calculations result in cumbersome relationships [7].

We consider a two-dimensional multilayer system with characteristics varying along one of the coordinate axes and heat-transfer processes described by the following system of equations:

$$\frac{\partial}{\partial x}\left[\varkappa(x)\frac{\partial T}{\partial x}\right]+\varkappa(x)\frac{\partial^2 T}{\partial y^2}=c(x)\frac{\partial T}{\partial t}, \tag{1}$$

$$\frac{\partial T}{\partial x}=0, \quad x=0, \tag{2}$$

$$-\varkappa\frac{\partial T}{\partial x}=\alpha(T-T_0), \quad x=L, \tag{3}$$

$$T(x,\ 0,\ t)=T_0, \quad T(x,\ M,\ t)=T_1, \tag{4}$$

$$T(x,\ y,\ 0)=T_0. \tag{5}$$

By introducing the variables

$$\Pi(x)=\int_0^x\frac{dx}{\sqrt{a(x)}}, \quad \Pi(y)=\int_0^y\frac{dy}{\sqrt{a(y)}}, \quad a=\frac{\varkappa}{c}, \tag{6}$$

and by assuming that the function $a(x)^{1/2}$ weakly depends on the x coordinates, i.e., the WCB approximation, we obtain:

$$\frac{\partial^2 T}{\partial\Pi^2(x)}+\frac{\partial^2 T}{\partial\Pi^2(y)}=\frac{\partial T}{\partial t}. \tag{7}$$

In relations (1)-(7) $\varkappa$, c, and α are the thermal conductivity, volume heat capacity, and heat-transfer coefficient; L and M are the system dimensions along the coordinates x and y. The boundary condition (3) is transformed into

$$-\frac{\partial T}{\partial\Pi(x)}=\frac{\alpha\sqrt{a(x)}}{\varkappa(x)}(T-T_0), \ \Pi(x)=\Pi(L). \tag{8}$$

Successive application of the finite integral Fourier sine transformation with respect to the variable $\Pi(y)$ and the eigenfunction method [9] to the initial equation set according to Eqs. (6)-(8) eventually gives

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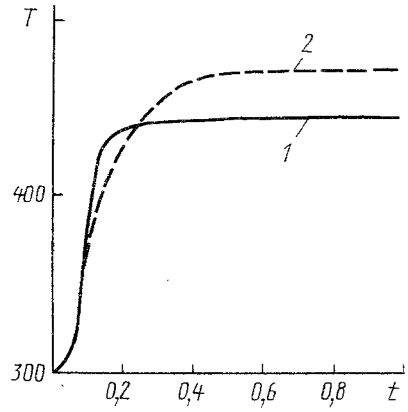

Fig. 1. The time dependence of temperature at the interface of a two-dimensional, two-layer plate (y = 0.25M): 1) numerical computer solution of the initial system; 2) calculation according to the relations (9), (10). T, K; t, sec.

$$T(x,\ y,\ t) \simeq T_0 + \frac{4}{\Pi(M)}(T_1 - T_0)\left\{\sum_{k=1}^{\infty} \frac{(\mu_k^2 + B_i^2)\sin\mu_k \cos\mu_k \frac{\Pi(x)}{\Pi(L)}}{\mu_k[\mu_k^2 + B_i^2 + B_i]} \times \right.$$

$$\times \left[\frac{\Pi(M)}{2}\,\frac{\mathrm{sh}\,\mu_k \frac{\Pi(y)}{\Pi(L)}}{\mathrm{sh}\,\mu_k \frac{\Pi(M)}{\Pi(L)}} + \frac{\pi}{\Pi(M)} \sum_{n=1}^{\infty} \frac{(-1)^n \sin \frac{n\pi\Pi(y)}{\Pi(M)}}{\frac{n^2\pi^2}{\Pi^2(M)} + \frac{\mu_k^2}{\Pi^2(L)}} \times \right. \tag{9}$$

$$\left.\left. \times \exp\left(-\frac{n^2\pi^2 t}{\Pi^2(M)} - \frac{\mu_k^2 t}{\Pi^2(L)}\right)\right]\right\},$$

where μ_k are the roots of the dispersion equation

$$\mu\,\mathrm{tg}\,\mu = B_i,\quad B_i = \frac{\alpha\sqrt{a(L)}\,\Pi(L)}{\varkappa(L)}. \tag{10}$$

Figure 1 compares the time dependence of a two-dimensional, two-layer plate at the interface $x = L_1 = L/2$ at $y = 0.25M$, obtained from relation (9), and that, obtained by computer calculation of the initial set of equations, $M = L = 10^{-2}$ m; $a_1/a_2 = 10^2$; $\varkappa_1/\varkappa_2 = 10^2$; $\varkappa_1 = 10^2$ W/(m·K); $c_1/c_2 = 1$; $a_1 = 10^{-4}$ m²/sec; $T_1 = 1000$ K; $T_0 = 300$ K; $B_i = 55$.

The implicit dependence of $\Pi(y)$ on x must be taken into account in the calculation.

A comparison of these results shows that the proposed procedure may be recommended for evaluation of an unsteady temperature field in two-dimensional multilayer systems.

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